1. A rook stands on the lower left square of a chessboard. Is there a path that takes the rook through every square of the chessboard once and only once, and that ends at the upper right square?

2. Find positive integers $x_1, x_2, \ldots, x_{1999}, x_{2000}$ such that

$$
\begin{align*}
1 - & \frac{1}{2^+} - \frac{1}{3^+} - \frac{1}{4^+} - \cdots - \frac{1}{1999} - \frac{1}{2000} = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_{1999}} + \frac{1}{x_{2000}}
\end{align*}
$$

3. Find all integer solutions of the equation

$$x^3 = 6y^3 + 20z^3.$$

4. Three circles with the same radius and centers $O_1$, $O_2$ and $O_3$ intersect at a given point $A$. Let $A_1$, $A_2$ and $A_3$ be the other intersection points. Prove that $\Delta O_1O_2O_3$ is congruent to $\Delta A_1A_2A_3$.

5. Write an essay of 400 to 600 words (complete with bibliography) on an application of mathematics to the study of the environment.