QUESTIONS:

1. You are standing in the center of a square room. Mirrors cover all four walls. Prove that you cannot see the back of your head. [Technical restatement: a light ray leaving the center point cannot next return to the center point from the diametrically opposite direction. Assume a light ray that strikes a corner reverses direction.]

2. Are there any natural numbers that can be represented both as the product of two and also four consecutive natural numbers?

3. In a convex quadrilateral $ABCD$, sides $AB$ and $CD$ are each divided into five equal segments, and corresponding points are joined as in the figure. Prove that the area of the center section $EFGH$ equals one-fifth of the area of $ABCD$.

4. Suppose there are $2^n$ balls in several baskets. An allowable operation consists of choosing two baskets, say $A$ with $p$ balls and $B$ with $q$ balls, $p \geq q$, and moving $q$ balls from $A$ to $B$. Show that after a finite number of allowable operations all of the balls can be brought to one basket.

5. Write an essay of 500 to 700 words (complete with bibliography) on an application of mathematics to cryptography.