STUDENT PRIZES
One 1st place prize • $300
Five 2nd place prizes • $150 each
Ten 3rd place prizes • $100 each

A full four-year academic tuition scholarship will be awarded to the first place prizewinner. In order to receive the scholarship, the winner must be directly admitted to the Purdue School of Science at IUPUI, major in any discipline in the School of Science, and attend full-time.

Scholarships in the amount of $2,000 per year will be awarded to the remaining cash prizewinners who are directly admitted to the Purdue School of Science at IUPUI and attend full-time. These scholarships are renewable for four years, given satisfactory academic performance. Honorable mentions will receive a gift. All entrants will receive certificates honoring their participation.

MATHEMATICS DEPARTMENT AWARDS
Schools awarded the 1st place trophy in previous years:

2012 • Carmel High School
2011 • Brownsburg High School
2010 • Carmel High School
2009 • Park Tudor High School
2008 • Carmel High School
2007 • Hamilton Southeastern H.S.
2006 • Carmel High School
2005 • Carmel High School

Spirit Award from previous years:
2012 • Avon High School

CEREMONY
All participants will be invited to an awards ceremony at IUPUI on Saturday, April 13, 2013 from 1:00-3:30 p.m. Parents and teachers will also be invited. The program will feature refreshments, a special presentation by John Goodpaster, Director of Forensic and Investigative Sciences Program, Department of Chemistry and Chemical Biology, and will end with the award presentation.

ELIGIBILITY
This contest is open to all Indiana high school students (grades 9-12).

Co Chairs: Jeffrey Watt, Roland Roeder

Special thanks to Bill Cross and Drazen Petrovic for submitting questions.

Contact Information:
www.math.iupui.edu/events/contest
IUPUI High School Mathematics Contest
Department of Mathematical Sciences
405 North Blackford Street, ID 270
Indianapolis, IN 46202-3216
(317) 274-6917 or contest@math.iupui.edu

ENTRIES: Mail your entry by Friday, March 8, 2013 to the address listed under Contact Information. You may obtain a copy of the questions, instructions for entering, and the cover page from your math teacher or the contest website: www.math.iupui.edu/events/contest. Solve the questions, giving your reasoning, not just the answers. Entries will be judged by professors in the IUPUI Department of Mathematical Sciences. Judging will be based on elegance of solution as well as correctness.

QUESTIONS

1) The digits from the set (1, 2, 3, 4, 5, 6, 7, 8, 9) are put into a hat and pulled out sequentially and without replacement to form a 9-digit number. What is the probability that the number formed is divisible by 11? Prove that your answer is correct.

2) Parts of a regular 5-gon ABCDE have areas denoted by \(x, y, \) and \(z\), as shown in the figure. If the area of \(x\) is given, compute \(y, z\), and the area of the whole regular 5-gon.

3) Let ABCD be the unit square, let \(S\) be the circle inscribed in ABCD, and let \(P\) be any point on the circle \(S\). Consider the line segments PA, PB, PC, and PD. Let \(a\) be the angle between PA and PC and \(b\) the angle between PB and PD. Show that \(\tan^2a + \tan^2b = 8\).

4) At a high school science competition, one of the events is the “cantilever”. Each team consists of 4 students, each of whose mass is any positive real number, with the sum of the 4 student’s masses adding up to at most 400kg. The idea is to use 3 weightless boards hanging over the stage, which are each exactly 4 meters long and to arrange the students to sit on those boards. For safety’s sake, only one student is allowed to be off of the stage.

The team’s score is the product of the weight of the student that is not on the stage times the distance of this student from the stage. However, if the structure tips, the team scores nothing.

You may assume that each of the students is a point mass and that the students and boards may be placed at arbitrary real/fractional positions.

Determine the maximum possible score and explain the configuration of students.

For example, suppose there were only two 4-meter boards and only three students with everything arranged as follows:

A, B, and C represent the students. Note that in order for the structure not to tip, \(C + 1/3B \geq A\) because otherwise the bottom plank would tip off the stage. Similarly, because A and B are equidistant from the end of the bottom plank, \(B \geq A\).

So under the restrictions of the problem, the most weight the team could get past the stage with this configuration is 180kg \((C = 100kg, B = 150kg, A = 150kg)\). The team’s final score would be 180 (kg) * 8 (m) = 1440.

5) Write an essay of 500 to 700 words (complete with references) on an application of mathematics to forensic science.